GLITCHES INDUCED BY THE CORE SUPERFLUID

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ABSTRACT

The long-term evolution of the relative rotation of the core superfluid in a neutron star with respect to the rest of the star, at different radial distances from the rotation axis, is determined through model calculations. The core superfluid rotates at a different rate (faster, in young pulsars), while spinning down at the same steady-state rate as the rest of the star, because of the assumed pinning between the superfluid vortices and the superconductor fluxoids. We find that the magnitude of this rotational lag changes with time and also depends on the distance from the rotation axis; the core superfluid supports an evolving pattern of differential rotation. We argue that the predicted change of the lag might occur as discrete events which could result in a sudden rise of the spin frequency of the crust of a neutron star, as is observed at glitches in radio pulsars. This new possibility for the triggering cause of glitches in radio pulsars is further supported by an estimate of the total predicted excess angular momentum reservoir of the core superfluid. The model seems to offer also resolutions for some other aspects of the observational data on glitches.

Subject headings: stars: neutron - hydrodynamics - pulsars

1. INTRODUCTION

Glitches are observed in radio pulsars as sudden changes $\Delta\Omega_c$ in the rotation frequency Ω_c of the crust with observed values of the jump in the range $10^{-9} \lesssim \frac{\Delta\Omega_c}{\Omega_c} \lesssim 10^{-6}$. In younger pulsars the jump in Ω_c is also accompanied by an increase $\Delta\dot{\Omega}_c$ in the observed spin-down rate $\dot{\Omega}_c$ of the crust which causes a recovery or relaxation back towards the pre-glitch behavior of Ω_c over time scales of days to years (Radhakrishnan & Manchester 1969; Lyne 1995). It is generally understood that glitches should be caused by mechanisms related to the internal structure of the star. This is because no correlated variation in the electromagnetic signature (intensity, polarization, pulse profile, etc.) of a pulsar has been observed at the time of their glitches. The two generally accepted mechanisms for glitches thus invoke starquakes (Baym

et. al. 1969) and "unpinning" of the vortices of a superfluid component in the *crust* (Anderson & Itoh 1975). In the latter mechanism, which is more relevant to the present discussion, a sudden release and rapid outward motion of a large number of otherwise pinned vortices acts as the source of the excess angular momentum which is transferred to the crust hence causing the observed jump in Ω_c . Suggested mechanisms for the sudden release of a large number of initially pinned vortices include catastrophic unpinning due to an intrinsic instability, breaking down of the crustal lattice by magnetic stresses, and thermal instability resulting in an increase in the mutual friction between the vortices and the superfluid (Anderson & Itoh 1975; Ruderman 1976; Greenstein 1979; Jones 1991; Link & Epsein 1996; Ruderman, Zhu & Chen 1998).

The core superfluid, on the other hand, is not commonly considered to play any major role in driving the glitches; there has been some earlier attempts in this regards which does not seem to have gained much support and acceptance (Packard 1972; Muslimov & Tsygan 1985; see, however, Sedrakian & Cordes 1997 for a recent suggestion based on a different model of pinning in the core than that invoked here). The coupled evolution of the neutron vortices and the proton fluxoids has been nevertheless discussed in various other respects, including its role in the post-glitch relaxation and also in driving glitches indirectly through crustal effects (Sauls 1989; Srinivasan et al 1990; Jones 1991; Chau, Cheng & Ding 1992; Ruderman et al 1998). Our aim here is to point at a so far neglected property of the rotational evolution of the core superfluid that might serve to cause glitches, directly. This is suggested based on the calculated long-term evolution of the rotational lag ω between the superfluid and its vortices; the latter being in corotation, on time scales larger than a few seconds, with the crust. Section 2 starts with a quick description of the model computations that were detailed elsewhere (Jahan-Miri 2000). In Section 2.1 the results of the same computations, generalized to the various superfluid shells within the stellar core, is used to advance the "jumping-lag" model as a new possibilty for the glitches. The model is further shown in Section 2.2 to be supported by the data on the unrelaxed components of the glitches. In Section 3 some other expected features of the glitch activity in pulsars is explored, based on a more detailed picture of the predicted evolution for the rotation of the core superfluid.

2. EVOLVING PATTERN OF SUPERFLUID DIFFERENTIAL ROTATION

The starting point for the new results, reported here, is the computations that were originally aimed at a study of the evolution of magnetic fields of pulsars (Ding et al. 1993; Jahan-Miri 2000). This required a determination of the velocity v_p of the outward motion of the fluxoids which carry the magnetic flux being

initially trapped within the superconducting core of the star. As a by-product, the superfluid rotational lag ω had also to be calculated simultaneously; although it had no immediate use for that study, it does so in a study of the superfluid rotation as attempted here. We adopt the same formalism, detailed in the above references, and generalize it to the various radial locations within the stellar core.

The steady-state radial motion of a fluxoid (the vortex of the proton superconductor in the core) at any given radial distance r from the rotation axis is determined from the balance equation (the Magnus equation) for all the radial forces that act on it. The forces include a pinning force F_n exerted by the vortices on fluxoids at their crossing points, as well as an outward buoyancy force F_b , an inward drag F_v due to the scattering of electrons, and an inward curvature F_c force which might operate at some stages. The effective value of the pinning force F_n , per unit length, on the fluxoids is decided by the Magnus force on the neutron-vortices, which is in turn determined by the rotational lag ω between the vortices and the neutron superfluid. Notice that ω represents also the difference between the rotational frequencies of the superfluid and that of the crust, which is the quantity of interest in the following discussion. The magnitude of F_n depends also on further assuming either of the following two plausible possibilities for the behaviour of vortices. Namely, a (neutron) vortex might remain straight and moves as a whole throughout its length, or else its individual pinned segments might move independently. The first is assumed in the model used here; see Jahan-Miri 2000 for the other possibility.

The magnitude of the pinning force which is exerted, at each intersection, by a vortex on a fluxoid, and vice versa, is however limited by a maximum value $f_{\rm P}$ corresponding to the given strength of the pinning energy $E_{\rm P}$ and the finite length scale $d_{\rm P}$ of the interaction, namely $E_{\rm P} = f_{\rm P}d_{\rm P}$. The Magnus force on the vortices which has to be balanced by the pinning force on them cannot therefore exceed a corresponding limit which in turn implies also a maximum (absolute) value for the lag ω . This maximum critical lag $\omega_{\rm cr}$ is estimated (Ding et. al 1993) as

$$\omega_{\rm cr} = 1.6 \times 10^{-10} B_c^{1/2} \text{ rad s}^{-1},$$
 (1)

where B_c is the strength of the magnetic field in the stellar core, in units of G. The dependence on B_c in the above equation is because of the dependence of ω_{cr} on the number density of the fluxoids, which would reduce as the flux continues to be expelled from the core. More explicitly, the pinning force per unit length of a vortex is inversely proportional to the spacing between the fluxoids, and consequently decreases as the number of fluxoids is reduced. Since the pinning force on the vortex, exerted by the fluxoids, is balanced by the Magnus force on it, exerted by the superfluid, a reduction in the former implies a reduction in the latter, ie. the Magnus force which is in turn proportinal to the lag $\omega_{\rm cr}$.

The equation of motion of the fluxoids is thus

$$F_{\rm n} + F_{\rm v} + F_{\rm b} + F_{\rm c} = 0.$$
 (2)

The forces are calculated from the following equations:

$$F_{\rm n} = 5.03 \times 10^{14} \frac{\omega}{P_{\rm s} B_8} \text{ dyn cm}^{-1}$$
 (3)

$$F_v = -7.30 \times 10^7 v_p \text{ dyn cm}^{-1}$$
 (4)

$$F_{\rm b} = 0.51 \, \text{dyn cm}^{-1}$$
 (5)

$$F_{\rm c} = -0.35 \, \text{dyn cm}^{-1}$$
 (6)

Substituting in Eq. 2 it reduces to

$$\alpha \frac{\omega}{P_{\rm s}B_{\rm c}} - \beta v_{\rm p} + \delta = 0, \tag{7}$$

where $P_{\rm s}$ is the rotation period in units of s, and α , β , and $\delta (\equiv F_{\rm b} + F_{\rm c})$ are the constants defined by Eqs 3–6. The effect of the curvature force $F_{\rm c}$ depends on further assuming fluxoids may bend when their radial velocity exceeds certain limiting value, $v_{\rm max}$, set by the speed of Ohmic diffusion of the magnetic field in the crust. Alternatively, it might be argued that collective rigidity of fluxoids requires them to remain always straight hence their velocity may never exceed the limiting speed of their end points; $v_{\rm p} \lesssim v_{\rm max}$. The first possibility has been assumed in the computations reported here, thus a negative inward curvature force, as given in Eq. 6, acts only when $v_{\rm p} > v_{\rm max}$; otherwise a value of $F_{\rm c} = 0$, hence $\delta = F_{\rm b}$, is used. Accordingly, δ attains only positive values, throughout the evolution time, for the adopted model; negative values of δ may be also realized for the other above choice of the effect of $F_{\rm c}$, nevertheless it does not lead to any significant new result for the purpose of the present discussion. Notice however that ω , in units of rad s⁻¹, might take either positive or negative values at different times, while α , and β are positive constants.

The above equation, Eq. 7, represents the azimuthal component of the Magnus equation of motion for the fluxoids and includes two unknown variables ω and v_p , in units of cm s⁻¹. It may be noted that the dependence on ω is through the Magnus force acting on the vortices, and that on v_p through the viscous drag force on the fluxoids. There exist however additional restrictions on the motion of the fluxoids which helps to fix, instantaneously, the value of one of the two variables and solve for the other, given the spin-down torque on the star which determines $\dot{\Omega}_s(t) \equiv \dot{\Omega}_c(t)$ and thus the radial velocity v_n of the vortices.

Namely, for a co-moving state $v_{\rm p}=v_{\rm n}$ is given and Eq. 3 can be be solved for ω . And, for the other two alternative cases where $v_{\rm p}$ is unknown and either $v_{\rm p}< v_{\rm n}$, or $v_{\rm p}>v_{\rm n}$ then ω is given as $\omega=\omega_{\rm cr}$, or $\omega=-\omega_{\rm cr}$, respectively. It should be however appreciated that one and only one of the above three solutions (viz., $v_{\rm p}=v_{\rm n},\,\omega=\omega_{\rm cr}$, or $\omega=-\omega_{\rm cr}$) would be satisfied at any time, for any given set of values of the variables $v_{\rm n},B_{\rm c}$, and $P_{\rm s}$.

Hence, from the instantaneous value of the spin-down rate of the star one finds the velocity v_n of the outward motion of the vortices, at any given instant, and for any assumed distance r from the axis. At the same time, ω_{cr} may be determined for the known value of the core field strength B_c , from Eq. 1. It is important to note that the superfluid is assumed to be spinning down at the same rate as the rest of the star, in order to find its long-term steady-state behaviour. A long-term permanent difference between the two rates would result in a permanent increase of the lag ω , which is ruled out. However, the true instantaneous value of the superfluid spin-down rate could as well be different than its time averaged value, that we have used. The instantaneous value of the spin-down rate may rather be determined by simultaneously solving the equations of motion of both the fluxoids as well as the vortices, for the given external torque on the star, which is not attempted here. In other words, the present calculations does not have the required time resolution (over time scales of a few years, or less) to fix the exact value of the lag ω at any given instant. The predicted values of ω should be thus understood as its averaged values, over time scales of a few years or so, denoted by ω_{∞} (to use the common notation).

A solution of Eq. 3, for the given values of $v_{\rm n}$ and $\omega_{\rm cr}$ at a given time t, then determines the corresponding values of the fluxoids velocity $v_{\rm p}$ and the lag ω_{∞} , at a given r. The predicted time behavior of ω_{∞} , as well as the critical lag $\omega_{\rm cr}$, for a single pulsar subject to the standard dipole torque, are shown in Fig. 1a & b, for the two locations $r=R_{\rm c}$ and $r=\frac{R_{\rm c}}{10}$, respectively, where $R_{\rm c}$ is the radius of the core of a neutron star. Three distinctive phases of relative rotation are realized, at each location, during the lifetime of the star, as is seen in Fig. 1. The lag takes positive and negative values, both with a magnitude $|\omega_{\infty}| = \omega_{\rm cr}$, during the early and the late times of the star's lifetime, respectively, and undergoes through the intermediate values at the intermediate times. During these three successive phases the radial motion of the vortices is, faster, same, and slower than the fluxoids, respectively. It is noted that when the fluxoids move (radially) faster, than the vortices, the pinning force on them is radially inward, ie. $F_{\rm n} < 0$, which corresponds to a negative value of the lag, $\omega_{\infty} = -\omega_{\rm cr}$. The superfluid in the core of a neutron star may thus spin down while maintaining a negative rotational lag with its vortices; an unusual state of affairs peculiar to the core superfluid. The above time variation of ω_{∞} (Fig. 1) is however superimposed on that

of $\omega_{\rm cr}$ itself which decreases steadily, as the magnetic flux is expelled out of the core (Eq. 1). As already indicated, similar results were reported earlier in studies of the magnetic evolution of pulsars (Ding et al 1993; Jahan-Miri 2000), which were however restricted to the single location of $r = R_{\rm c}$, as required for that purpose.

One can observe, from Fig. 1, that as a neutron star ages there is a secular change in the rotational lag ω_{∞} between the core superfluid and the "crust," which is true for both radial locations indicated in that Figure. At both radii (and in fact throughout the core, as will be further demonstrated, below) the magnitude of ω_{∞} decreases over the lifetime of the star, and becomes finally vanishingly small around an age $\sim 10^7$ yr. This is indeed a manifestation of the expulsion of the magnetic flux out of the core, during that time period. The core field strength B_c determines (see Eq. 1) ω_{cr} which in turn sets the limit on the magnitude of ω_{∞} . Moreover, a comparison of the two panels in Fig. 1 reveals that $\omega_{\rm cr}$ differes by an order of magnitude between the two radii indicated. This comes about simply because of the r-dependence of the Magnus force on the vortices that in turn determines the pinning force on the fluxoids (see Eqs 2 & 3 in Jahan-Miri 2000). Thus the core superfluid must be "rotating differentially," ie. its spin frequency is a function of the distance r from the rotation axis; $\Omega_{\rm s}(r)$. Fig. 1 points at still another property of the superfluid rotation, namely the change in ω_{∞} occurs first at the inner radii and later at the outer radii. That is, the pattern of differential rotation of the core superfluid evolves over the lifetime of a pulsar. This is further demonstrated explicitly in Fig. 2 where the predicted relative value of Ω_s with respect to Ω_c (taken as the reference value) is plotted as a function of r. It is appreciated that the Figure has been produced by calculating the same quantity $\omega_{\infty}(r,t)$, at each time and location separately, and using the relation $\omega = \Omega_s - \Omega_c$. The two panels of Fig. 2 correspond to two extreme cases considered: (a) a relatively young pulsar, with an age comparable to the Crab pulsar, and (b) a relatively old pulsar with an age $\geq 10^7$ yr. The dash-dotted line in Fig. 2b (to be further exploited below) represents the loci of the largest negative values (of ω_{∞}) attained at different intermediate times at the different corresponding values of r. As may be seen from Fig. 2b (the dotted line), the initial pattern of (differential) relative rotation has disappeared and also the superfluid, at all values of r, has come into near corotation with the rest of the star (the "crust"), by an age $\sim 10^7$ yr. The excess angular momentum associated with the initial state of rotation, in contrast to the final corotation, must have obviously been shared between the superfluid and the "crust," while both have been also spinning down at the same steady-state rate, driven by the radiation torque acting on the star.

2.1. The "Jumping-lag" Model

The important question that now remains to be addressed is whether the above predicted change in the rotational lag, corresponding to the transition from conditions (between the dotted lines) in Fig. 2a to that in Fig. 2b, occurs in a gradual and continuous manner or instead as discrete jumps. In other words whether the superfluid rotation remains always in a stable equilibrium state or else the predicted long-term evolution of its spin frequency (as well as ω_{∞}) would be hindered among successive metastable states, superimposed on its secular spinning down. If the latter is indeed the relevant process one would then expect that the sudden relaxation events in ω_{∞} would produce corresponding discrete jumps in the spin frequency of the crust. As already indicated, the predicted evolution of ω (Fig. 1) is based on the assumed condition $\dot{\Omega}_{\rm s} = \dot{\Omega}_{\rm c}$, which is mandatory only for the time-averaged value of the superfluid spin-down rate. However, the instantaneous value of $\dot{\Omega}_s$, which determines v_n for a solution of Eq. 7, may as well be different than its steady-state value thus calculated. The computed curve of time evolution of ω in Fig. 1 represents but an "envelope" for its true time-resolved behaviour that could as well depart from the one given here. As was pointed out long ago by Packard (1972) and Anderson & Itoh (1975), it is quite likely that the pinned vortices in a neutron star exhibit metastable equilibrium states, similar to that observed in the laboratory experiments of superfluid Helium (Tsakadze & Tsakadze 1975). The pinned core superfluid might as well evolve through metastable states which would make the predicted reduction in ω_{∞} , hence in $\Omega_{\rm s}$, to occur discontinuously. That is the pinned superfluid in the core may spin-down at a rate smaller than that of the crust (its container) over short time scales (of, say, a few years or less) which will result in a temporary build up of the value of ω , as is observed in laboratory experiments and is also invoked for the crustal superfluid. The situation is however unstable and a sudden relaxation will restore the dynamically predicted value of ω_{∞} , which is itself evolving over the much larger time scale of the evolution of the star. Should the change in the lag, or part of it, be instead accomplished in a smooth gradual way then its observable effect would be a decrease in the so-called braking index n of pulsars $(n = \frac{\Omega_c \ddot{\Omega}_c}{\dot{\Omega}_c^2})$. We emphasize that the above two possibilities for the behaviour of ω are at an equivalent footing as far as the present calculations can say; here we will explore the consequences of one of the two alternatives at a greater length.

The predicted magnitude of ω_{∞} in the core (see Fig. 1) is, on the other hand, large enough such that an assumed sudden relaxation of the superfluid, or part of it, could safely account for a jump in $\Omega_{\rm c}$ similar to that observed in the glitches. The largest observed glitches, in Vela pulsar, accompany a change $\Delta\Omega_{\rm c} \sim 10^{-4} \ {\rm rad \ s^{-1}}$ in the rotation frequency of the "crust." The crust would include, for the present model, the remaining part of the stellar moment of inertia except that in the core superfluid which is the

donor of the angular momentum that would cause a glitch. Thus a change $\Delta\Omega_{\rm s} \sim 10^{-5}$ rad s⁻¹ in the rotation frequency of the core superfluid would be required, considering a ratio $\sim 10\%$ for the fractional moment of inertia of the "crust." The required change $\Delta\Omega_{\rm s}$ is seen, from Fig. 1, to be much smaller than the predicted values of ω_{∞} (note, also, the values of ω_{∞} in Fig. 3b, below, which are even larger); a relaxation of only part of ω_{∞} , in only some regions of the superfluid, could quantitatively be responsible for the largest observed glitches.

The above possibility for the driving cause of glitches, based on the long-term evolution of the core superfluid, may be contrasted with the other suggested mechanisms that are likewise based on a superfluid component of the star. In the most popular model glitches are driven by a sudden relaxation of the crustal superfluid out of an assumed metastable state (Anderson & Itoh 1975). A similar mechanism had been also suggested for the pinned core superfluid (Packard 1972). In both these models, the instantaneous value of ω is assumed to initially differ from its expected steady-state value, hence the superfluid being in a temporary metastable state. The relaxation, that manifests as a glitch, then serves to reduce the instantaneous lag to achieve its steady-state value (it might overshoots to lower values as well). Thus, the magnitude of the steady-state lag plays no role by itself in these models; what matters is the departure of the instantaneous lag from its steady-state value. The latter could be however, in those models, a preserved quantity, in principle; it varies slightly only owing to the long-term variation of, the external torque hence, the spin-down rate of the whole star. In contrast, in the model that is proposed here the evolution of the steady-state lag itself drives the effect; here this is a quantity which would vary even if the spin-down rate of the whole star were to remain constant! In still another model for the glitches the pinned core superfluid is assumed to remain decoupled from the secular spinning down of the "crust" most of the time, viz. throughout the inter-glitch intervals (Muslimov & Tsygan 1985). Hence, the lag increases steadily untill $\omega_{\rm cr}$ is reached which causes a sudden spin-down of the superfluid, hence a decrease in ω . This is again different than the jumping-lag model wherein the core superfluid is assumed to continuously take part in the steady-state spinning down of the star, at a rate similar to the "crust." Therefore, the alternative or the additional mechanism that is suggested here for the glitches, relying on an assumed pinned superfluid component in the stellar core, differs from the earlier similar models in a fundamental way, too. The jumps in the rotational lag, suggested here, are a consequence of the evolution of the steady-state lag itself, that could *not* be avoided, either.

For the same reason, the new suggested model could in principle provide a predictable rate for the occurrence of the glitches; more accurately, an estimate for the time average of the rate times the magnitude

of the glitches. In contrast the catastrophic unpinning events suggested for the crust superfluid (Anderson & Itoh 1975) are unpredictable and stochastic in nature. According to the model of vortex creep (Alpar et. al. 1984) the crust superfluid might as well spin down at a given steady-state rate while the pinned vortices are creeping steadily, at a prescribed steady-state value of the lag. This situation should however persist indefinitely, according to that model, without leading by itself to the conditions required for the sudden unpinning events. Even though, for the glitch-inducing free movement of the vortices $\omega \gtrsim \omega_{\rm cr}$ is needed however the assumed steady-state conditions in the creep model does not lead to it directly; some other instabilities must be invoked. Therefore, the glitch expectancy rate calculated (Alpar & Baykal 1994) based on the vortex creep model seems to be contradicting the underlying creep process, particularly, because $\omega_{\infty} < \omega_{\rm cr}$ is assumed (Alpar et al. 1984). In contrast, the jumps in the lag suggested here are driven by the lifetime evolution of the spin period and the magnetic field of the star which would determine the average rate of change in ω_{∞} , hence the glitch expectancy rate. It is further noted that the predicted decrease in the value of the lag is a robust result, being independent of the particular choice of the forces acting on the fluxoids. Indeed, even in the absence of any other force except the pinning force the same is expected; the magnitude of the lag would still decrease because the number density of the fluxoids reduces as they migrate out of the core. True, in the latter case the lag would never become negative, however this does not change our arguments, in any fundamental way. Also, it would be appreciated that the realization of a pattern of differential rotation, or its particular profile, is not a necessary requirement for the core superfluid in order to play its role in driving the glitches; it may do so even if it were to rotate rigidly. The differential rotation, as indicated by our results, nevertheless adds to the diversity of the effects that might be expected to occur at glitches induced by the core superfluid.

Post-glitch relaxation ?!

Before proceeding further with the new model for inducing glitches, the post-glitch relaxation associated with the assumed pinned core superfluid might present itself as a major drawback for the model. The point is that given the large moment of inertia of the core, a disturbance in ω might be expected to cause a decoupling of the superfluid from the spin-down of the rest of the star, hence an increase in the observable spin-down rate much larger than is usually observed. Although a detailed discussion of the post-glitch reponse caused by the assumed pinning in the core superfluid is not intended here, its possible role in driving the glitches may not be however objected, on the above ground, for the following reasons. One may assume, for the sake of the present argument, that the core superfluid remains *coupled* throughout a glitch. This is possible, and consistent with the suggested mechanism for the glitches, because the quantity which

undergoes a jump is the steady-state value of the lag! there need not be a departure from it, hence no reason for a decoupling either. The spin-down rate of the superfluid has been indeed set, in the computations reported here, equal to the rest of the star, which is consistent with the above assumption. Moreover, the difficulty might arise if the core superfluid is assumed to decouple as a *whole*. A decoupling of only some part of it, which could be accommodated by the predicted evolving pattern of differential rotation, and/or a dynamically *partial* decoupling, could be consistent with the existing observational constraints. One is reminded that the very large post-glitch spin-down rates already observed would be in fact more consistent with a partial contribution from the core superfluid, than otherwise; fractional increase in the spin-down rate by $\sim 60\%$ in Vela, and > 10% in PSR 0355+54, over timescales of ~ 0.4 d, and ~ 44 d, respectively, has been observed (Lyne 1987; Flanagan 1995).

2.2. The Q-Test

A quantity of interest in the study of glitches, that may be determined observationally, is the recovery amplitude expressed in terms of a percentage recovery factor Q. The unrelaxed part of a glitch is parametrized as $(1-Q)\Delta\Omega_c$, with $0 \le Q \le 1$. The different observed values of the recovery factor in different glitching pulsars seem to be correlated with the age of the pulsar. In the younger pulsars the jump in Ω_c is accompanied by an increase in the spin-down rate $\dot{\Omega}_c$, both of which approach their extrapolated pre-glitch values over the post-glitch relaxation period $(Q \sim 1)$. For the older pulsars, on the other hand, observations show that there is very little recovery in $\Delta\Omega_c$ (Q << 1), and that the small amount of the recovery depends inversely upon the characteristic age of the pulsar (Lyne 1995).

A glitch caused by the proposed sudden relaxation of ω_{∞} to its current expected value may also offer a natural explanation for the observed large unrelaxed component of $\Delta\Omega_{\rm c}$. The distinction with other similar models lies in the fact that, as indicated earlier, the *pre-glitch* value of ω_{∞} need not be anymore recovered during the post-glitch era. That is, in the other models the rotational lag acts as a temporary reserve tank of angular momentum which is pumped in during the inter-glitch interval and out at the glitch. However in the new model the tank need not be re-filled to its previous level! The (1-Q) part of the glitch may correspond, in this scenario, to the net amount of relative angular momentum which ought to be lost by part of the superfluid, owing to the predicted long-term decrease in the value of $\omega_{\infty}(r)$, achieved instantaneously. The same would be deposited in the crust, which will show up as the offset above the pre-glitch extrapolated values of $\Omega_{\rm c}$. Nevertheless, zero values of Q could be also accomodated, since

simultaneous positive and negative jumps in ω_{∞} at different locations within the core could occur (see Fig. 3 below).

The above correspondence may however be tested against the existing observational data on the unrelaxed parts of glitches for pulsars of various ages. As is implied by Fig. 1a and Fig. 1b, a decrease in $\omega_{\infty}(r)$, at relatively early times, is predicted only for the inner regions of the core superfluid. At later times, however both the inner and the outer parts would contribute, noticing also the larger moment of inertia of the outer regions. Accordingly, a larger loss of angular momentum by the superfluid, hence smaller Q values, might be expected for the older pulsars, which is in agreement with the observed trend (Lyne 1995). Moreover, the total predicted *initial* excess angular momentum in the superfluid core of a young pulsar should be, according to this model, comparable to (or at least smaller than, to make allowance for other mechanisms) the total sum of that associated with the unrelaxed parts of all the glitches during the whole pulsar lifetime. This may be tested, quantitatively, against the data at hand. The observable accumulated increase $\Delta\Omega_{\rm total}$ in the rotation frequency of the crust due to the unrelaxed parts of all glitches occurring between times t_1 to t_2 could be estimated as

$$\Delta\Omega_{\text{total}} = \int_{t_1}^{t_2} (1 - Q(t)) \ \Omega_{\text{c}}(t) \ A(t) \ dt, \tag{8}$$

where the glitch activity parameter A(t) is the fractional increase in the rotation frequency (per year) due to the glitches, and t is in units of yr. Based on the observational data collected by Lyne (1995), the two unknown functions may be estimated as

$$Q(t) \sim 176 t^{-0.88}$$
, and (9)

$$A(t) \sim 4.87 \times 10^{-3} t^{-1.04},$$
 (10)

where the latter applies to times $t \gtrsim 10^4$ yr. Also, the observed periods of pulsars may be fitted as

$$\Omega_{\rm c} \sim 7000 \ t^{-0.5} \ \rm rad \ s^{-1}$$
 (11)

for an assumed standard dipole torque. Neglecting the contribution from times $t \leq 10^4$ yr, which is small because of the large values of Q for that period, Eq. 4 then implies a value of

$$\Delta\Omega_{\text{total}} \sim 0.3 \text{ rad s}^{-1},$$
 (12)

for all glitches expected during a pulsar lifetime. Therefore, and in order for the unrelaxed parts of the glitches to be associated with the predicted long-term decline of ω_{∞} during a pulsar lifetime, an average initial value of $\omega_{\infty} = \omega_{\rm cr} \sim 0.03 \text{ rad s}^{-1}$ would be expected, assuming a fractional moment of inertia for

the core superfluid $\sim 90\%$; a lower proportion might be however more plausible according to predictions of some equations of states for the matter in the core, which will increase the expected value of ω_{∞} . The above expected value of ω_{∞} may be contrasted with the results in Figs 1 or 2, which are seen to be of the same order of magnitude; they agree perfectly for the inner radii, though. The overall agreement might be considered to be promissing, given the theoretical uncertainties of the model calculations and the poor statistics of the data used to derive the expressions for A and Q. However, a better agreement is already within the reach. Namely, the values of $\omega_{\rm cr}$ in Figs 1 & 2 correspond to an assumed lower limit for the pinning energy $E_{\rm P} \sim 0.3$ MeV. The adopted value of $E_{\rm P}$ could as well be larger by a factor of $\gtrsim 6$; since $\frac{\lambda}{\xi} = \sqrt{2}$ has been used (following Eq. 7 in Ding et. al 1993) instead of $\frac{\lambda}{\xi} \sim 10$ for the proton superconductor in the core, which brings in the factor $\frac{\ln 10}{\ln \sqrt{2}} \sim 6$ in the dependence of $E_{\rm P}$ on $\ln \frac{\lambda}{\xi}$. The larger corresponding values of $\omega_{\rm cr}$ may be read off from Fig. 3b, below. Thus the model seems to account for all, or at least the major part, of the observed unrelaxed components of the glitches. It may be recalled that according to the vortex creep model the unrelaxed parts of the glitches are assumed to be caused by a so-called "capacitor region" in the crust superfluid that remains permanently pinned and decoupled from the rest of the star except for its effect at a glitch (Alpar 1995). The suggested resolution here for the unrelaxed parts of glitches might at least serve as a complementary process next to the formation of capicitor regions, since the latter has been found inadequate to account, by itself, for all the observed effect (Lyne 1995).

3. Other Implications

As we have argued, the profile of $\Omega_{\rm s}(r)$ across the stellar core, relative to $\Omega_{\rm c}$, is different at different ages of a pulsar, because the transition from the initial to the final values shown in Fig. 2 occurs at different epochs for the different regions (Fig. 1). Moreover, in each region, $\Omega_{\rm s}$ is predicted to first drop to a value (the dash-dotted line in Fig. 2b) smaller than its corresponding final value at times $t > 10^7$ yr (the dotted line in Fig. 2b). The predicted relative profiles at intermediate phases are shown in Fig. 3 (the thick dashed line), where the initial and final profiles are also plotted (the dotted lines) for reference. Fig. 3 shows, in effect, the transition between the conditions of Fig. 2a to that in 2b in smaller time steps; it presents few snap-shots from the long-term evolution of the pinned core superfluid, identifying some characteristically different phases which may be realized. Figs 3a & 3b are identical, except that a larger value of the pinning energy ($E_{\rm P} = 1.8~{\rm M~eV}$) has been used in Fig. 3b, which results in the larger values of ω_{∞} , too. As was indicated, these larger values of ω_{∞} (Fig. 3b) could produce a better agreement between the predicted value of the integrated glitch activity ($\Delta\Omega_{\rm total}$) with that implied by the observations. Moreover, Fig. 3b

indicates a lower age limit for the youngest pulsars in which a glitch might be induced by the core superfluid (ie. $\sim 10^3$ yr, in contrast to values $\sim 3 \times 10^4$ yr implied by Fig. 3a), as judged by the earliest time when a departure from the initial values of ω_{∞} is seen to occur.

One can see, from Fig. 3, that the discrete jumps in $\omega_{\infty}(r)$ could behave differently, hence having different observable effects, depending on the age of the pulsar. The expected jump at different times may occur in different regions, with different magnitudes, and also with opposite signs, with an effect that would be weighted by the moment of inertia of the region(s) involved. The observable effect would, moreover, depend on the way in which the released excess angular momentum is shared between the crust and the rest of the core superfluid, as noted earlier. An attempt to draw a definite, and quantitative, correspondence between theory, at the present status of the model, and observations could not be, admittedly, conclusive. However, the potentially rich consequences of the model should be emphasized. In the following we briefly list some of the features that might be expected, based on the results in Fig. 3, for the jumps at different stages of a pulsar lifetime, and speculate on their possible observational effects.

- The sudden change in the rotation rate of the core superfluid occurs initially only at the innermost regions. Because of the smaller moment of inertia of these regions the corresponding induced jumps in the rotation rate of the crust (glitches) might be expected to be smaller in the youngest pulsars.
- At later times the changes should take place in larger regions, hence the glitch activity should increase with the pulsar age.
- However, the innermost regions of the core-superfluid are also expected to soon reach the stage of makeing *negative* transitions from their earlier states corresponding to the *dash-dotted* line in Fig. 2b to the less negative values of the lag.
- Later on, in the oldest pulsars, the inner regions would have reached their final evolutionary states and only the outer parts would be responsible for the glitches.
- The competition between these effects should result in a peak for the glitch activity, as well as the magnitude of the glitches, to occur at some intermediate pulsar age.
- Negative jumps in the rotation frequency of old pulsars, with an age $t_{\rm sd} \gtrsim 10^7$ yr, might be observed when ω_{∞} is negative throughout the core and its magnitude is decreasing in order to achieve the final predicted values. As already indicated, the transition between the initial and the final patterns of superfluid rotation shown in Fig. 2 (the dotted lines) occurs after an earlier transition to states with

lower (relative) angular momenta, ie. the more negative values of ω , also indicated in Fig. 2 (the dash-dotted line). Accordingly, as a normal glitch (ie. a sudden spin-up of the crust) would correspond to transitions accompanying a decrease in the (algebraic) value of ω , likewise a transition in which ω increases its value (jumping from the dash-dotted line toward the dotted line in Fig. 2b) may result in an (observable) sudden spin-down of the crust.

• A "glitch" may or may not show up as a *sudden* rise in the observable rotation rate of the crust. That is to say the "standard" *fast-rising* glitches observed usually may be a subset of a larger class of disturbances in the rotation of a neutron star, induced by its core superfluid. If the excess vortices released at an event originate from an outer region of the core superfluid they might move out, and annihilate at the core-crust boundary, fast enough to induce a "standard" fast-rising glitch. However, should the released vortices belong to an inner region then the required re-arrangement of all vortices in the core, in order to transfer the excess angular momentum out to the crust, might be suspected to be hindered by the existing pinning barriers. The process might as well take some macroscopically large time; a quantitative estimate would require a model for the coupling of the superfluid under conditions different than the assumed steady-state. A "slow rise" in the angular velocity, over a period of few days, has been already observed in the youngest known pulsar, ie. the Crab (Lyne 1995). It is interesting to note that the predicted jump in the lag for the *youngest pulsars* also occurs only in the *innermost regions*.

To summarize, it was observed that the superfluid component in the core of a neutron star, being subject to the pinning of its vortices with the fluxoids, rotates differentially. The pattern of the superfluid differential rotation evolves with time owing to an accompanying decrease in the number of the fluxoids which are expelled out of the core. Over time scales of millions of years the superfluid lag diminishes and the differential rotation almost disappears. Namely, the superfluid rotation initially supports a reservoir of excess angular momentum in the core of a neutron star which is exhausted as the star evolves. Thus it should act as an additional *spin-up* mechanism for the rest of the star, the "crust", superimposed on the overall spinning down of the both, which is driven by the general dipole torque acting on a neutron star. It was suggested that the predicted decrease in the lag, hence in the spin frequency of the superfluid, might take place at discrete steps in which case it would show up as the observed glitches in radio pulsars. Some aspects of the observational data on glitches were argued to be consistent with, and support, the predictions made on this basis. Further improvement, and test, of the model requires simulation of the long-term evolution of the fluxoid-vortex motion to be carried out, self-consistently, across the entire core.

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Fig. 1.— Time evolution of the steady-state value of the superfluid rotational lag ω_{∞} and the critical lag $\omega_{\rm cr}$, at a distance (a) $r = R_{\rm c}$, and (b) $r = 0.1R_{\rm c}$, from the ortation axis, where $R_{\rm c}$ is the radius of the core of a neutron star. Note the difference in scales for ω between (a) and (b).

Fig. 2.— Relative profile of the angular velocity of the core superfluid (dotted line) with respect to that of the "crust" (full line) for a (a) very young, and (b) very old pulsar, as denoted by its age t_{sd} . The dash-dotted line in (b) corresponds to the largest negative values of the lag achieved at different intermediate times during the lifetime of the star, for the different r-values.

Fig. 3.— Relative profiles of the the angular velocity of the core superfluid (thick dashed line) with respect to that of the "crust" (full line), at the different times marked on each graph, in units of years. The dotted lines, above and below the full line show (for reference) the initial values of Ω_s and its largest negative relative values (same as the dash-dotted line in Fig. 2b), respectively. (a) and (b) are identical, except that a larger value of the pinning energy has been assumed in (b).











